The Role of Color Neutrality in Nuclear Physics- Modifications of Nucleonic Wave Functions

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Abstract

The influence of the nuclear medium upon the internal structure of a composite nucleon is examined. The interaction with the medium is assumed to depend on the relative distances between the quarks in the nucleon consistent with the notion of color neutrality, and to be proportional to the nucleon density. In the resulting description the nucleon in matter is a superposition of the ground state (free nucleon) and radial excitations. The effects of the nuclear medium on the electromagnetic and weak nucleon form factors, and the nucleon structure function are computed using a light-front constituent quark model. Further experimental consequences are examined by considering the electromagnetic nuclear response functions. The effects of color neutrality supply small but significant corrections to predictions of observables.

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I. INTRODUCTION

Nucleons are composite color singlet systems made of quarks and gluons. Their wave functions consist of many configurations as illustrated in Fig.1. Some configurations are simple with only three current quarks, most are more complex with many partons. Configurations in which three quarks are close together have been dubbed point-like configurations (PLC). [1] According to perturbative quantum chromodynamics (pQCD), such configurations are responsible for high momentum transfer elastic scattering reactions [2,3]. Furthermore, the effects of gluon emission from closely separated color-singlet systems of quarks and gluons tend to cancel. This means that for processes in which one adds amplitudes before squaring, i.e. coherent processes, point like configurations do not interact with surrounding media.

That color neutrality can suppress interactions is similar to the cancellation of interactions which would occur if an electron and positron would move together at the same position through a charged medium (see for example Ref. [4]). The color neutrality feature of QCD has been verified. It is responsible for the scaling of structure functions at low, but not too low, values of the Bjorken scaling variable x_{Bj}. (See the reviews [5–7].) Furthermore hadron-proton total cross sections σ_{hp} grow linearly with the mean square radius of the hadron [8].

There are also configurations in the nucleon wave function in which the partons occupy a larger than average size. We call these configurations blob-like configurations BLC or huskyons [9]. These configurations have complicated strong interactions with the medium. (See [9] which explores the consequences of huskyons.) Here we model the interaction of the nucleon with the surrounding medium in terms of the inter-quark separation within the nucleon to investigate the role of color neutrality and its consequences in nuclear wave functions.

Sixty years of studies of nuclear properties have shown that the nuclear wave function is dominated by clusters of color singlet objects with the quantum numbers of nucleons and

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mesons. The success of the nuclear shell model is a testimony to this fact. It is certainly possible, however, that although bound nucleons largely maintain their identity, bound and free nucleons are not identical. Indeed, there are a number of interesting experimental findings which may indicate that medium modifications of nucleon properties are relevant.

One of the most spectacular examples is the observation of the first EMC effect [10] showing that the structure function of the bound nucleon was suppressed at large x_{Bj}. There have also been numerous studies of the (e, e') reaction which find that the longitudinal response function R_L is suppressed, while the transverse response R_T is not. See Ref. [11] and references therein. The suppression of R_L is natural in some theories [12]. This lore has been challenged recently by Jourdan [13] who argues that the "so-called quenching is mostly due to the limited significance of the data" and that including data at high energy loss ω leads to the result that "no A-dependent quenching is observed". However, the various errors listed in Jourdan's Table 2 allow for up to about 15% effects.

It has also long been stated that the nuclear value of the axial coupling constant G_A was less than its value in free space [14]. Furthermore the large pionic enhancement expected in many models of nuclear structure was not observed [15], but there may be hints of a pionic enhancement [16].

Thus there are several indications that nucleons could be modified by the medium. Since color is at the heart of QCD, one might suspect that looking at the consequences that color neutrality might have for bound composite nucleons might be worthwhile. These are the presumably small effects of nucleonic polarization. Nevertheless, as we show here, these effects can have nontrivial consequences. Before describing in detail our treatment of the nucleon in the medium, it is necessary to mention some of the possible objections and also to discuss some of the relevant history.

The most significant objection is that there could be many effects other than color neutrality which cause significant modifications of the properties of nucleons. This is certainly a valid point. For example, in the Walecka model [17] the medium provides a decrease of the nucleon mass by as much as 30%. See also the quark-meson coupling model of Ref. [18],

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and the works of Ref. [19,20]. It is entirely conceivable that similar effects to the nucleon form factors could occur from a purely hadronic mechanism, i.e., independent of the relative quark separation. It has also been argued that a more general scaling of hadronic masses may arise in the nuclear medium. [21] The extension of QCD sum rules to finite density further confirms the presence of large scalar and vector self-energy corrections [22,23]. Nevertheless, these effects have a different origin than that of color neutrality, and our aim here is to simply study the consequences of this single effect in several different situations by identifying a pattern of predictions. The study of many reactions is necessary, so we derive a formalism applicable to many reactions. This formalism may be useful in the study of other mechanisms for medium modifications as well as other corrections to the impulse approximation.

Another objection is that it is not clear when pQCD, and hence its discussion of point-like configurations is applicable. The question of how high the momentum transfer must be for pQCD considerations to dominate calculations of form factors is controversial. The work of Refs. [24,25] is merely the first example of a long history. More data are needed to settle this question. However, point-like configurations may arise from non-perturbative considerations as well [26,27]. The singular nature of the non-perturbative confining interaction can lead to point-like configurations. Indeed, many constituent quark and Skyrmion models seem to indicate the presence of point-like configurations. It is therefore of interest to consider an interaction with the nuclear medium which accomodates such objects.

Another possible objection is the failure to observe clear and convincing evidence for color transparency CT. This is the idea that a high Q^2 quasielastic reaction produces a point-like configuration which does not interact with the surrounding nuclear medium. In this way initial and/or final state interactions are reduced. A BNL experiment [28] shows some evidence for CT in a (p, pp) experiment at beam momenta of 6, 10 and 12 GeV/c (4.5 GeV^2 < Q^2 < 8 GeV^2) but the $(e, e'p)$ NE18 ($Q^2 = 1, 3, 5, 7$ GeV^2) experiment at SLAC [29] sees no such evidence. A recent FNL exclusive ρ production experiment saw evidence for color transparency [30]. The Q^2 was up to about 9 GeV^2. The BNL and FNL experiments

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may not have guaranteed the necessary quasielastic nature of the experiment. The SLAC experiment did that. One interpretation of these results is that point-like configurations are produced but expand before leaving the nucleus. This expansion is a bigger effect for the NE18 experiment, which has the lowest energy outgoing protons. Expansion is also a big effect for the BNL experiment, but the incident proton has high enough momentum for this expansion not to completely kill the influence of the point-like configurations. The expansion effects are smallest for the FNL experiment, but the experimental resolution may be a problem.

None of the considerations of the above paragraphs are sufficient to rule out the existence of point-like configurations, or the effects of color neutrality in nuclear physics. Indeed, point-like configurations were introduced in the context of the EMC effect [1]. Frankfurt and Strikman postulated that such configurations do not feel the attractive nuclear interaction and therefore appear in a smaller percentage in nuclei than in free space. This leads to a reduction of the nuclear structure function at $x_{Bj} \sim 0.4 - 0.6$. This effect also reduces the cross sections predicted for $(e, e'p)$ reactions, making color transparency harder to observe [31]. Frankfurt and Strikman treat their effect by taking the suppression to vary essentially as a theta function in Q^2, i.e., present at Q^2 above a certain value but completely absent for lower values. Desplanques finds a similar term by including the spatial variation of the nuclear scalar and vector mean-fields over the volume of the nucleon [32]. Desplanques's treatment was non-relativistic. Note also that Jain and Ralston [33] have argued that hard processes are modified significantly by medium effects.

Our aim here is to present a method for doing calculations which may be applied both at high and low values of Q^2. We also obtain a Lorentz covariant formulation. Here is an outline. Our model and basic formalism are presented in the next section. Section 3 deals with some simple examples which allow us to study our approximations and expose the need for a relativistic formalism. The relativistic formalism, which applies the recent work of Schlumpf [34–36], is discussed next. Section 5 is concerned with a presentation of our detailed numerical results. We study medium modifications of the electromagnetic

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proton form factors G_E and G_M and the axial vector form factor G_A relevant in the weak interaction. We discuss consequences in the e, e' and $e, e'p$ reactions. A concluding section discusses and asseses these results.

II. FORMALISM AND MODEL

We begin from general considerations. Suppose a color singlet baryon moves in the nucleus subject to a Hamiltonian H given by

$$H = H_0 + H_1 \tag{1}$$

where H_1 is a perturbing Hamiltonian, and

$$H_0 = H_0^{cm} + H_0^{rel}. \tag{2}$$

Here H_0^{cm} and H_0^{rel} describe respectively the motion of the center of mass (including effects of the medium on the center-of-mass motion), and the relative motion of the internal degrees of freedom independent of the medium and the center-of-mass motion. The perturbation H_1 describes mixing between the center-of-mass and relative motion, and thereby incorporates the effects of the medium on the internal wave function.

We shall assume that H_1 is separable, that is,

$$H_1 = H_1^{cm} h_1^{rel}, \tag{3}$$

where H_1^{cm} and h_1^{rel} act only in the center-of-mass and relative motion spaces respectively. Our separation of the Hamiltonian in (1) and (2) entails that h_1^{rel} requires a change in the internal structure of the baryon. This implies that h_1^{rel} has no diagonal elements, and that the baryon in the medium is described as a superposition of excitations.

We wish to consider the eigenstates $|\Psi_{nm}>$ of the hamiltonian H in (1), defined by

$$H|\Psi_{nm} >= E_{nm}|\Psi_{nm} > . \tag{4}$$

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The form of H_0 in (2) allows the unperturbed states to be written as a direct product of center-of-mass and relative motion states as

$$|\Psi_{nm}^{(0)}> = |\Phi_n>|\phi_m>. \tag{5}$$

Here $|\Psi_{nm}^{(0)}>$ satisfies the eigenvalue equation

$$H_0|\Psi_{nm}^{(0)}> = E_{nm}^{(0)}|\Psi_{nm}^{(0)}>, \tag{6}$$

where $H_0^{cm}|\Phi_n> = \xi_n|\Phi_n>$ and $H_0^{rel}|\phi_m> = \epsilon_m|\phi_m>$, with $E_{nm}^{(0)} = \xi_n + \epsilon_m$.

The state vector of the full Hamiltonian can be written to first order in H_1 as

$$|\Psi_{nm}> = |\Psi_{nm}^{(0)}> + \sum_{kl \neq nm} \frac{<\Psi_{kl}^{(0)}|H_1|\Psi_{nm}^{(0)}>}{E_{nm}^{(0)} - E_{kl}^{(0)}}|\Psi_{kl}^{(0)}>. \tag{7}$$

To simplify our notation we restrict our discussion to the nuclear ground state. For this case Eq.(7) can be rewritten as

$$|\Psi_{00}> = |\Psi_{00}^{(0)}> + \sum_k |\Phi_k><\Phi_k|H_1^{cm}|\Phi_0> \sum_{l \neq 0} \frac{<\phi_l|h_1^{rel}|\phi_0>}{E_{00}^{(0)} - E_{kl}^{(0)}}|\phi_l>. \tag{8}$$

It should be noted that in (8), since h_1^{rel} has no diagonal elements, $l \neq 0$ and the sum on k is unrestricted. The energy denominator in Eq.(8) is dominated by nucleonic excitation energies. These, which are typically hundreds of MeV, are much larger than the nuclear energy differences which are typically tens of MeV. Thus we write

$$E_{00}^{(0)} - E_{kl}^{(0)} \approx \epsilon_0 - \epsilon_l. \tag{9}$$

This allows the sum on k to be performed using completeness so that

$$|\Psi_{00}> = \left[|\phi_0> + \sum_{l \neq 0} \frac{<\phi_l|h_1^{rel}|\phi_0>}{\epsilon_0 - \epsilon_l}|\phi_l> H_1^{cm}\right]|\Phi_0>. \tag{10}$$

We define the quantity in brackets as $|\tilde{\phi}>$

$$|\tilde{\phi}> \equiv |\phi_0> + \sum_{l \neq 0} \frac{<\phi_l|h_1^{rel}|\phi_0>}{\epsilon_0 - \epsilon_l}|\phi_l> H_1^{cm}, \tag{11}$$

which can be regarded as the modified nucleonic wave function. Note that this wave function depends on the coordinates of the entire nucleus through the operator H_1^{cm}. The use of

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Eq.(11) allows one to evaluate the effects of color neutrality for finite nuclei; this equation is our principle formal result.

The evaluation of Eq.(11) depends on knowing the wave functions corresponding to the Hamiltonian H_0^{rel}. At the present time there is no complete relativistic treatment available, but progress has recently been made in this direction [37]. We wish to obtain an alternate method of evaluating the influence of the nuclear medium on nucleonic wave functions. Such can be obtained using a closure approximation in which all of the strength is assumed (on average) to lie at an average excitation energy. We use $\epsilon_0 - \epsilon_l = \Delta E$. The expectation value of an operator $\mathcal{O}$ can then be written to first order in H_1 as

$$< \tilde{\phi}|\mathcal{O}|\tilde{\phi} >=< \phi_0|\mathcal{O}|\phi_0 > +\frac{H_1^{cm}}{\Delta E} < \phi_0| \left\{ h_1^{rel}, \mathcal{O} \right\} |\phi_0 >, \qquad (12)$$

where $\{A, B\} = AB + BA$. The right side of (12) is independent of the excited state wavefunction, and can therefore be evaluated based on knowledge of the ground state wavefunction and the excitation energy. Thus the uncertainty in the knowledge of the wave functions is replaced by the uncertainty in a single parameter, ΔE, and by the further assumption that ΔE does not depend very strongly on the operator $\mathcal{O}$, i.e. that ΔE is essentially process independent. The accuracy of this approximation and assumption is investigated in section 4. We shall discuss reasonable values of ΔE after discussing our choice of the operator h_1^{rel}.

Equation (12) has a simple physical interpretation. Consider, for example, the case where the operator $\mathcal{O}$ is the electromagnetic(EM) current j_μ associated with a constituent of the state $|\phi_0 >$. A modification of the form factor will occur by the mechanism illustrated in Fig.2. The current acting on the ground state excites an internal mode. The excitation decays back to the ground state via the interaction with the medium provided by h_1^{rel}. The extent to which the form factor is modified is determined by the ability of the EM current to produce an excitation and the ability of the medium to absorb its decay and return the composite particle to its ground state. The modification is directly accountable to the density of the medium in the vicinity of the interaction through the potential, $H_1^{cm}(R)$, and is suppressed by the excitation energy ΔE.

Medium effects are often estimated by taking the nucleus to be infinite nuclear matter, or by using a local density approximation. Eq.(12) shows that instead one can evaluate the appropriate nuclear matrix which depends on H_1^{cm}.

III. COLOR SCREENING

Consider, now the motion of a composite color singlet baryon through a nucleus. We are here interested in the properties of the ground and low lying nuclear states, so that the baryon is part of a bound state wave function. The configurations of the baryon are pictured in Fig.1. Let the displacement of the center of mass of the baryon from the nuclear center be denoted as $\vec{R}$. The interactions between such a complicated system and the remainder of the nucleus must depend on the positions $\vec{r}_i$ of the partons inside the baryon.

In general the interactions between a nucleon and the rest of the nucleus are complicated. Here we are studying the presumably small effects of nucleonic polarization, so we concentrate on the necessary modifications of the central part of the nuclear shell model potential. This is the largest interaction to consider, and a small modification of it might have significant consequences. Thus we have the central potential $V = V(\vec{r}_i, \vec{R})$. What else do we know? Color neutrality tells us that $V = 0$ when $\vec{r}_i = \vec{R}$ for all partons i. Furthermore interactions vanish as r^2, where $r^2 = \Sigma_{i<j}(\vec{r}_i - \vec{r}_j)^2$ [38–40]. Finally we note that the nucleonic average over $\vec{r}_i$ should correspond to the standard nuclear shell model potential. These considerations allow us to write

$$V(r, R) = V_0 \rho(R) \frac{r^2}{<r^2>}, \tag{13}$$

where $<r^2> \equiv <\phi_0|r^2|\phi_0>$ is the nucleonic expectation value of the operator r^2, $V_0 \approx -50\text{MeV}$ and $\rho(R)$ is the nuclear density normalized so that $\rho(R=0)=1$.

We are concerned with fluctuations, so that it is convenient to rewrite the central shell model potential as

$$V(r, R) = V_0 \rho(R) + V_0 \rho(R) \frac{r^2 - <r^2>}{<r^2>}, \tag{14}$$

so that the operator H_1 of previous sections can be identified as

$$H_1 = V_0\rho(R) \ \frac{r^2 - <r^2>}{<r^2>}. \tag{15}$$

This is the simplest form of H_1 that we can write, which is consistent with known properties. The behavior for large values of r^2 is simply a guess, but it is reasonable to expect that large, blob-like configurations should have strong interactions with the medium.

The equation (13) includes the effects of color neutrality on the total nuclear mean field and makes no distinction between the scalar and vector mean fields. The effects of color neutrality on the separate fields can be examined using the quark-meson coupling model [18], but this would require the extension of the applicability of that model to high values of the transferred momenta.

We note that H_1 has the interesting property that

$$< \phi_0|H_1|\phi_0 >= 0, \tag{16}$$

which means that the color neutrality effect governed by our perturbing Hamiltonian does not give a first-order shift in the mass of the baryon. Such effects are by definition contained in the first term of Eq.(14).

Let us discuss some of the implications before presenting the evaluation of specific models. Firstly, consider the value of ΔE. We see that the operator $r^2 - <r^2>$ excites the breathing mode of the nucleon. Thus it is reasonable to associate ΔE with the the energy at which the first resonance of the nucleon occurs. This is the Roper resonance with ΔE=-500 MeV. We are concerned with how certain matrix elements are influenced by the presence of the nuclear medium, so it is convenient to define $\delta < \mathcal{O} >\equiv< \tilde{\phi}|\mathcal{O}|\tilde{\phi} > - < \phi_0|\mathcal{O}|\phi_0 >$. Then using Eq.(15) in Eq. (12) leads to

$$\delta < \mathcal{O} >= 2 \ \frac{V_0\rho(R)}{\Delta E} < \phi_0|\mathcal{O} \ \frac{r^2 - <r^2>}{<r^2>}|\phi_0 > . \tag{17}$$

This shows that the nucleon's properties as measured by $\delta < \mathcal{O} >$ depend on the position of the nucleon. The factor $2V_0\rho(0)/\Delta E$ is about +0.2, which shows that the effects we study here are small but not insignificant.

It is useful to consider examples. First suppose $\mathcal{O} = r^2$. Then

$$\delta < r^2 >= 2 \; \frac{V_0 \rho(R)}{\Delta E} < \phi_0| \; \frac{r^4 - < r^2 >^2}{< r^2 >} |\phi_0 > . \tag{18}$$

Writing $r^4 = r^2 \sum_n |\phi_n >< \phi_n| r^2$, subtracting the ground state contribution, and examining the remainder and using completeness shows that $\delta < r^2 > \; > 0$. Color neutrality leads to an increase in the mean square radius of the nucleon. Note that the same argument can not be used to show that $\delta < r^4 > \; > 0$; indeed this quantity could be negative. Color neutrality does not correspond to a general scaling of the nucleon wave function. Each matrix element must be worked out independently.

Another example to consider occurs in the non-relativistic quark model if $\mathcal{O} = \mu \equiv \sum_{i=1,3} \mu_3(i)$. In this case the expectation value of $\mathcal{O}$ for a spin up nucleon is the magnetic moment of free nucleon. In the non-relativistic quark model the wave function is a product of space and spin-isospin functions. In this case, the integral over the spatial coordinates vanishes and $\delta < \mu >= 0$. Similarly, in the non-relativistic quark model the modification to the electric and magnetic form factors must be the same. It is necessary to extend these considerations to relativistic models.

It is very interesting to consider the case when $\mathcal{O} = \mathcal{O}_0 = \delta[(\vec{r}_1 - \vec{r}_2)/\sqrt{2}]\delta[(\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3)/\sqrt{6}]$ for which all of the quarks are at the same position. The expectation value of $\mathcal{O}_0$ is the square of the wave function at the origin. In this case the assumption that $V(r = 0, R)$ vanishes is sufficient to give a result, i.e., no additional assumption about the dependence on r^2 for large r^2 is needed. One finds immediately (using Eq.(17) for example) that

$$\delta < \mathcal{O}_0 >= -\frac{2V_0 \rho(R)}{\Delta E} |\phi_0(0)|^2 , \tag{19}$$

which corresponds to about a 20% reduction of the square of the wave function at the origin. This result is in good agreement with a recent QCD sum rule calculation by Jin et al. [41].

It is worthwhile to compare our approach with that of Frankfurt and Strikman [1]. Those authors consider the nucleon $|N >$ as a sum of various configurations, $|N >= |PLC > + \cdots$. At high Q^2 the form factors are assumed to be dominated by the $|PLC >$ component. In

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the medium $|PLC>$ is replaced by $(1+\frac{H_1}{\Delta E})|PLC>$. However, $r^2|PLC>=0$, so that the PLC acquires a factor of $(1-\frac{V_0\rho(R)}{\Delta E})$. This leads to $10-20\%$ reductions in form factors, if Q^2 is greater than some value large enough for $|PLC>$ to dominate. This result is recovered in our approach if we assume that the only dependence of the form factors on the (modification of the) nucleon wave function is through the wave function at the origin. Even more striking is the feature that the $|PLC>$ is suppressed, so that one expects valence quarks to carry less momentum in the nucleus. This leads to an explanation of the EMC effect. Our approach is motivated by the original work of Frankfurt and Strikman. However, we do not make the "all or nothing " assumption in which $|PLC>$ suppression turns on abruptly. Thus we are concerned with making more detailed evaluations than and testing the assumptions of the early work. In particular, we wish to learn if these suppression effects persist if one uses more detailed models.

IV. TOY MODELS

To gain some insight into the results obtained for the nucleon in the relativistic framework to follow in section 5, it is useful to apply the formalism of the previous two sections to some simple non-relativistic constituent quark models involving (mainly) only two quarks. These simplifications enable us to obtain exact solutions and therefore to assess the validity of our approximation scheme. We shall start by using the harmonic oscillator model. However, such a model is not expected to be a reasonable guess for large momentum transfers. Indeed previous work [26,27] showed that the harmonic oscillator does not have a point-like configuration. Therefore we also consider the case of two quarks bound by a Coulomb potential. This model assumes that the attractive nature of the color electric force dominates over the longer range confining force. Finally we consider a model with both a harmonic confinement and Coulomb term as in [26,27].

A. Harmonic Oscillator

Here we start by considering the "nucleon" to be made of two quarks, and employ the nonrelativistic constituent quark model with H_0^{rel} given by the harmonic oscillator form

$$H_0^{rel} = \frac{p^2}{2\mu} + \frac{1}{2}\mu\omega_0^2 r^2. \tag{20}$$

We study the nuclear medium-modified nucleon wave function $\tilde{\phi}$ at $R = 0$. This is given by

$$\left[\frac{p^2}{2\mu} + \frac{1}{2}\mu\omega_0^2 r^2 + V_0\rho(0)\frac{r^2}{<r^2>}\right]\tilde{\phi} = E\tilde{\phi}. \tag{21}$$

We may easily obtain the exact solution by realizing that in the medium the frequency ω_0 is replaced by ω with

$$\omega^2 = \omega_0^2 + \frac{2V_0\rho(0)}{\mu<r^2>}$$
$$= \omega_0^2 + \frac{4}{3}V_0\rho(0)\omega_0. \tag{22}$$

For typical values of $\Delta E = -2\omega_0 \sim -500$ MeV and $V_0 = -50$ MeV, we find that $\omega = 214$ MeV and the expectation value of r^2 increases by a ratio of 1.17. So, at the nuclear center, the effect of color neutrality can be significant. The expectation value of r^{-1} varies as the inverse of the square root of the frequency, so that its expectation value is decreased by a factor of 0.93.

We may also compute the form factor for this model. This is the Fourier transform of the square of the wave function as a function of the momentum transfer $Q = |\vec{Q}|$ divided by two:

$$F_0(Q^2) = <\phi_0|e^{i\vec{Q}\cdot\frac{\vec{r}}{2}}|\phi_0> \tag{23}$$

$$F(Q^2) = <\tilde{\phi}|e^{i\vec{Q}\cdot\frac{\vec{r}}{2}}|\tilde{\phi}>. \tag{24}$$

The free form factor $F_0(Q) = e^{-\frac{Q^2}{16\mu\omega_0}}$, while the medium modified form factor $F(Q) = e^{-\frac{Q^2}{16\mu\omega}}$. We see that

$$\lim_{Q^2\to\infty} \frac{F(Q^2)}{F_0(Q^2)} = 0, \tag{25}$$

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so that huge effects at high Q^2 are possible. However, this model is not realistic at high Q^2.

We may also consider the effects of color neutrality for very dense nuclear systems by increasing $\rho(0)$ above its value of unity in normal nuclear matter. Eq.(22) tells us that ω vanishes for densities about four times nuclear matter density for which $\rho(0) = \frac{3\omega_0}{4V_0} = 3.75$. Thus there is a "deconfinement" phase transition inherent in our model. However, the model is built on the assumption that the shell model, with its non-overlapping nucleons, is a valid starting point and therefore should not be applied to the situation for which the interparticle spacing is less than the diameter of a nucleon.

We note that the extension of this model to the 3-quark system yields essentially the same results. This is because the harmonic interaction $r^2 = \sum_{i<j}^{3}(\vec{r}_i - \vec{r}_j)^2 = 3(\rho^2 + \lambda^2)$ where $\vec{\rho} = (\vec{r}_1 - \vec{r}_2)\sqrt{2}$ and $\vec{\lambda} = (\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3)/\sqrt{6}$. Using the same operator in H_1 once again leads to the result that for harmonic oscillator models the frequency in the nuclear medium is less than that in free space.

Let us examine the accuracy of our first order (fo) treatment of Eq.(17). In first order $\delta < r^2 >_{fo} = \frac{-2V_0\rho(0)}{3\omega_0} < r^2 >$ compared with a model exact result of $\delta < r^2 >= \left((1 + \frac{4V_0\rho(0)}{3\omega_0})^{-1/2} - 1)\right)$. With our typical parameter, the first order increase in the mean square radius is 13%, while the exact result is a shift of 17%.

The EM form factor in the medium obtained from Eq.(12) is given by

$$F(Q^2) = \left[1 - 2\frac{V_0}{\Delta E}\frac{Q^2}{6\mu\omega_0}\right]F_0(Q^2), \tag{26}$$

where $F_0(Q^2) = \exp(-Q^2/16\mu\omega_0)$ is the free form factor. Eq.(26) compares favorably with the exact solution with corrections of order $(V_0/\Delta E)^2$. Some caution should be exercised, however, in the application of the approximate solution to large momenta, i.e., $Q^2 \sim 6\mu\omega_0\Delta E/V_0$, where the coefficient of the $(V_0/\Delta E)^2$ correction becomes significant. It is also important to note that the interaction with the medium has produced an increase in the mean square radius, $< r^2 >\approx r_0^2[1 + \frac{4}{3}(V_0/\Delta E)]$, as is reflected in both the exact and the approximate form factor.

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B. Coulomb binding

The harmonic oscillator is not expected to describe situations involving high momentum transfer. Indeed we may recall the asymptotic expansion:

$$\lim_{Q^2 \to \infty} F_0(Q^2) \approx \frac{-8\pi}{Q^4} \frac{d\phi_0^2}{dr}\bigg|_{r=0} + \frac{16\pi}{Q^6} \frac{d^3}{dr^3}\phi_0^2\bigg|_{r=0}. \tag{27}$$

This power-law dependence gives larger results than the Gaussian form and holds unless the potential is an analytic function of r^2. The only relevant example of such is the harmonic oscillator force of the previous sub-section. It is useful to obtain the general expression for the medium-induced change in the form factor $\Delta F(Q^2) \equiv F(Q^2) - F_0(Q^2)$. To first-order in H_1 this is given by

$$\Delta F(Q^2) = 2\sum_{n \neq 0} \frac{<\phi_0|e^{i\vec{Q}\cdot\frac{\vec{r}}{2}}|\phi_n><\phi_n|H_1|\phi_0>}{E_0 - E_n}. \tag{28}$$

One may then define the state vector $|\chi>$ such that

$$(E_0 - H_0^{rel})|\chi> = (1 - |\phi_0><\phi_0|)H_1|\phi_0>, \tag{29}$$

with $<\chi|\phi_0> = 0$. Then

$$\Delta F(Q^2) = 2 <\phi_0|e^{i\vec{Q}\cdot\frac{\vec{r}}{2}}|\chi>, \tag{30}$$

and

$$\lim_{Q^2 \to \infty} \Delta F(Q^2) = \frac{-8\pi}{Q^4} \frac{d}{dr}(\phi_0\chi)\bigg|_{r=0} + \frac{16\pi}{Q^6} \frac{d^3}{dr^3}(\phi_0\chi)\bigg|_{r=0}. \tag{31}$$

Our purpose here is to compare the exact first-order result of Eq.(29) with that of the closure approximation of Eq.(12).

The above equations are general, but we can gain some understanding if we specify to the Coulomb Hamiltonian

$$H_0^{rel} = \frac{p^2}{2\mu} - e^2/r, \tag{32}$$

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with $e^2 = 4\alpha_s/3$ to simulate the strong color electric force. The ground state wave function is given by

$$\phi_0(r) = \frac{1}{(\pi a_0^3)^{\frac{1}{2}}} e^{-\frac{r}{a_0}}, \tag{33}$$

with $a_0 = 1/(e^2\mu)$. The form factor for this state is

$$F_0(Q^2) = \left[1 + \frac{Q^2 a_0^2}{16}\right]^{-2}. \tag{34}$$

The Hamiltonian of Eq.(32) can be used in Eq.(29) to obtain a solvable differential equation. The result is the function $\chi(r)$ given by

$$\chi(r) = \frac{V_0 \rho(0) \mu a_0^{1/2}}{3\sqrt{\pi}} e^{-r/a_0} [11/2 - (r/a_0)^2 - (r/a_0)^3/3]. \tag{35}$$

One may use Eqs.(31) and (35) to immediately find that, at large values of Q^2, $\Delta F(Q^2)$ varies as Q^{-4} and is negative. In this model, the high Q^2 form factor is dominated by the point like configuration, so we may say that the point like configuration is suppressed in the nuclear medium. It is not difficult to use Eqs. (30) and (35) to obtain an exact expression for $\Delta F(Q^2)$. However it is more instructive to use the asymptotic expansion of Eq.(27) to get $\Delta F(Q^2)$ as an expansion in powers of Q^{-2}. Then one finds

$$\lim_{Q^2 \to \infty} \Delta F(Q^2) = \frac{V_0 \rho(0) \mu}{3 \, a_0^2 \, Q^4} \left[88 - \frac{544}{Q^2 \, a_0^2}\right]. \tag{36}$$

Recall that V_0 is a negative quantity, so that ΔF is also negative. The closure approximation to the change of the form factor, $\Delta F_{clos}(Q^2)$ can be obtained from Eq.(12) as

$$\Delta F_{clos}(Q^2) = \frac{2V_0 \rho(0)}{<r^2> \Delta E}(-4\nabla_Q^2)F_0(Q^2), \tag{37}$$

which may be evaluated asymptotically as

$$\Delta F_{clos}(Q^2) \approx -\frac{8 \, V_0 \rho(0)}{3\Delta E} \left(\frac{16}{Q^2 \, a_0^2}\right)^3. \tag{38}$$

It is clear that the closure approximation can not be valid unless ΔE is taken to be a function of Q^2. We may equate the $\Delta F's$ of Eqs.(38) and (36) to determine the "correct" value of ΔE. The asymptotic result is

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$$\Delta E = \frac{-4096}{\mu a_0^2}[11Q^2a_0^2 - 68]^{-1}. \tag{39}$$

This means that the magnitude of the ΔE decreases as Q^2 increases, so that using the closure approximation with a fixed value of ΔE underestimates the effects of the medium modifications. This is the principal result of this subsection.

C. Harmonic Oscillator plus Coulomb

A more interesting model is one which includes both a confining and a Coulomb type $1/r$ term in the Hamiltonian. Thus we consider

$$H_0^{rel} = \frac{q^2}{2\mu} + \frac{1}{2}\mu\omega_0^2 r^2 - \frac{4}{3}\frac{\alpha_s}{r}. \tag{40}$$

This model was considered in Refs. [26,27] in which it was shown that including the Coulomb term leads to PLC dominance of the form factor. This occurs even though the Coulomb term causes only a small effect in the computed energy.

For the calculations performed here the parameters are taken to be $\alpha_s = 0.1$, $\mu = 300\text{MeV}/c^2$ and $\hbar\omega_0 = 390\text{MeV}$. Fig.3 shows how the ground state wave function in free space is influenced by the $1/r$ term in the Hamiltonian. The attraction enhances the wave function at the origin and changes the shape away from the Gaussian. This causes a $1/Q^4$ behavior in the form factor. Next, Fig. 4 compares the full wave function with and without the effects of the medium. We see that the medium causes a reduction of the short distance wave function. The closure approximation to the wave function in the medium is compared with the exact calculation in Fig. 5. The numerical results for the medium-modified form factor are shown in Fig.6 along with the closure result and the free form factor, from which one concludes that closure works well and in fact slightly underestimates the exact result. This is consistent with the conclusions of the previous subsection.

It is also useful to try to understand these results using analytic techniques. This can be done if one works to first order in α_s. First consider the free case. One may find the form

factor using an equation like Eq. (28), but with the operator H_1 replaced by $-4\alpha_s/3r$. The result is

$$F_0(Q^2) = e^{-z_0}\left(1 + \frac{4\alpha_s/3}{(\sqrt{\pi}/2)}\sqrt{\mu/\omega_0}f(z_0)\right). \tag{41}$$

with

$$f(z_0) = \sum_{n=1}^{\infty}\frac{1}{n}\frac{1}{2n+1}\frac{1}{n!}z_0^n, \tag{42}$$

where

$$z_0 \equiv Q^2/(16\mu\omega_0). \tag{43}$$

The second term of Eq.(41) leads to a $(\mu\omega_0)^2/Q^4$ behavior of the form factor which dominates over the exponential term for values of z_0 greater than two or so. Then the free form factor varies as $\omega_0^{3/2}$. The medium modification is quite easy to implement. Simply replace ω_0 by ω of Eq.(22)in the formulae (41)-(43). We have seen in Eq.(22) that $\omega = 0.86\omega_0$ so that the form factor is reduced by a factor of 0.79. This is a significant suppression. The results of this perturbative analysis are shown in Fig.7. Comparison with Fig.6 shows that the perturbative analysis is in good agreement with the full result.

V. RELATIVISTIC CONSTITUENT QUARK MODEL OF THE NUCLEON

The influence of medium modifications on computed matrix elements and form factors are examined using non-relativistic models in the previous sections. The need for a relativistic formulation is clear; we wish to compute form factors at momentum transfers $Q^2 > 1$ GeV^2, and we wish to be able to distinguish between electric and magnetic effects.

Here we study the effects of color neutrality on the light front nucleonic model wave functions of Schlumpf [34–36]. The first formulation of such a light front relativistic quark model was presented by Terent'ev and Beretskii [42,43]. Many authors [44–52,37,53–56,?] have contributed to the development of this model. We use Schlumpf's model because his

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power-law wave functions lead to a reasonably good description of the proton electromagnetic from factors, G_E and G_M, at all of the Q^2 where data are available [34–36].

In a quantum mechanical relativistic theory the commutation relations between the ten generators of the Poincar$\acute{e}$ group must be respected. The light front approach is distinguished by the feature that the maximal number of seven generators are of kinematical character (do not contain interactions). Another feature involves the use of the light front variables $p^{\pm} \equiv p^0 \pm p^3$, so that the Einstein mass relation $p_{\mu}p^{\mu} = m^2$ can be expressed as

$$p^- = ((p_{\perp}^2 + m^2)/p^+, \tag{44}$$

where $p_{\perp} \equiv (p^1, p^2)$. Susskind [58] noted that this equation is similar to the non-relativistic kinetic energy if one interprets the variable p^+ to be a relativistic version of the mass. Thus one can use relative momenta for systems involving several particles, with the result that the wave function is a simple product of a function involving only relative momenta with a separate function carrying information about the motion of the center of mass. One may also employ the Melosh transformation [59] to construct states that are eigenfunctions of the total angular momentum and its third component.

The light front dynamics have another important feature, stressed in Refs. [3,44], that the diagrams with quarks created out of or annihilated into the vacuum do not contribute. Furthermore, one need only consider three quark components of the nucleon, if one is computing the matrix elements of "good " operators [60].

Schlumpf's model is well-documented [34–36]. We reproduce the relevant features here for the sake of clarity, following his thesis closely. It is necessary to express the ten generators of the Poincaré group P_{μ} and $M_{\mu\nu}$ in terms of dynamical variables to specify the dynamics of a many-particle system. The kinematic subgroup is the set of generators that are independent of the interaction. There are five ways to choose these subgroups [61]. Usually a physical state is defined at fixed x^0, and the corresponding hypersurface is left invariant under the kinematic subgroup.

The light-front formalism is specified by the invariant hypersurface $x^+ = x^0 + x^3 =$

constant. The following notation is used: The four-vector is given by $x = (x^+, x^-, x_\perp)$, where $x^\pm = x^0 \pm x^3$ and $x_\perp = (x^1, x^2)$. Light-front vectors are denoted by an arrow $\vec{x} = (x^+, x_\perp)$, and they are covariant under kinematic Lorentz transformations [62]. The three momenta $\vec{p}_i$ of the quarks can be transformed to the total and relative momenta to facilitate the separation of the center of mass motion [63] as

$$\vec{P} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3, \quad \xi = \frac{p_1^+}{p_1^+ + p_2^+}, \quad \eta = \frac{p_1^+ + p_2^+}{P^+},$$
(45)

$$q_\perp = (1 - \xi)p_{1\perp} - \xi p_{2\perp}, \quad K_\perp = (1 - \eta)(p_{1\perp} + p_{2\perp}) - \eta p_{3\perp}.$$

Note that the four-vectors are not conserved, i.e. $p_1 + p_2 + p_3 \neq P$. In the light-front dynamics the Hamiltonian takes the form

$$H = \frac{P_\perp^2 + \hat{M}^2}{2P^+},$$
(46)

where $\hat{M}$ is the mass operator with the interaction term W

$$\hat{M} = M + W,$$
$$M^2 = \frac{K_\perp^2}{\eta(1 - \eta)} + \frac{M_3^2}{\eta} + \frac{m_3^2}{1 - \eta},$$
(47)
$$M_3^2 = \frac{q_\perp^2}{\xi(1 - \xi)} + \frac{m_1^2}{\xi} + \frac{m_2^2}{1 - \xi},$$

with m_i being the masses of the constituent quarks. To get a clearer picture of M we transform to q_3 and K_3 by

$$\xi = \frac{E_1 + q_3}{E_1 + E_2}, \quad \eta = \frac{E_{12} + K_3}{E_{12} + E_3},$$
(48)

$$E_{1/2} = (\mathbf{q}^2 + m_{1/2}^2)^{1/2}, \quad E_3 = (\mathbf{K}^2 + m_3^2)^{1/2}, \quad E_{12} = (\mathbf{K}^2 + M_3^2)^{1/2},$$

where $\mathbf{q} = (q_1, q_2, q_3)$, and $\mathbf{K} = (K_1, K_2, K_3)$. The expression for the mass operator is now simply

$$M = E_{12} + E_3, \quad M_3 = E_1 + E_2.$$
(49)

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The use of light front variables enables one to separate the center of mass motion from the internal motion. The internal wave function Ψ is therefore a function of the relative momenta $\mathbf{q}$ and $\mathbf{K}$. The function Ψ is a product $\Psi = \Phi \chi \phi$, with $\Phi = $ flavor, $\chi = $ spin, and $\phi = $ momentum distribution. The color wave function is antisymmetric.

The angular momentum $\mathbf{j}$ can be expressed as a sum of orbital and spin contributions

$$\mathbf{j} = i\nabla_\mathbf{p} \times \mathbf{p} + \sum_{j=1}^{3} \mathcal{R}_{Mj} \mathbf{s}_j \ , \tag{50}$$

where $\mathcal{R}_M$ is a Melosh rotation acting on the quark spins $\mathbf{s}_j$, which has the matrix representation (for two particles)

$$\langle \lambda' | \mathcal{R}_M(\xi, q_\perp, m, M) | \lambda \rangle = \left[\frac{m + \xi M - i\sigma \cdot (\mathbf{n} \times \mathbf{q})}{\sqrt{(m + \xi M)^2 + q_\perp^2}} \right]_{\lambda' \lambda} \tag{51}$$

with $\mathbf{n} = (0, 0, 1)$. The effects of the Melosh rotation are to significantly increase the computed charge radius [48].

The operator $\mathbf{j}$ commutes with the mass operator $\hat{M}$; this is necessary and sufficient for Poincaré-invariance of the bound state. In particular, $j^2 | \Psi, \uparrow \rangle = 3/4 | \Psi, \uparrow \rangle$ and $j_z | \Psi, \uparrow \rangle = 1/2 | \Psi, \uparrow \rangle$. The angular momentum operator is in terms of relative coordinates given by

$$\mathbf{j} = i\nabla_\mathbf{K} \times \mathbf{K} + \mathcal{R}_M(\eta, K_\perp, M_3, M)\mathbf{j}_{12} + \mathcal{R}_M(1 - \eta, -K_\perp, m_3, M)\mathbf{s}_3 \ ,$$

$$\tag{52}$$

$$\mathbf{j}_{12} = i\nabla_\mathbf{q} \times \mathbf{q} + \mathcal{R}_M(\xi, q_\perp, m_1, M_3)\mathbf{s}_1 + \mathcal{R}_M(1 - \xi, -q_\perp, m_2, M_3)\mathbf{s}_2 \ .$$

The orbital contribution does not contribute for the ground state baryon octet, so that

$$\mathbf{j} = \sum \mathcal{R}_i \mathbf{s}_i \ ,$$

$$\mathcal{R}_1 = \frac{1}{\sqrt{a^2 + K_\perp^2}\sqrt{c^2 + q_\perp^2}} \begin{pmatrix} ac - q_R K_L & -aq_L - cK_L \\ cK_R + aq_R & ac - q_L K_R \end{pmatrix} \ ,$$

$$\mathcal{R}_2 = \frac{1}{\sqrt{a^2 + K_\perp^2}\sqrt{d^2 + q_\perp^2}} \begin{pmatrix} ad + q_R K_L & aq_L - dK_L \\ dK_R - aq_R & ad + q_L K_R \end{pmatrix} \ , \tag{53}$$

$$\mathcal{R}_3 = \frac{1}{\sqrt{b^2 + K_\perp^2}} \begin{pmatrix} b & K_L \\ -K_R & b \end{pmatrix} \ ,$$

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with

$$a = M_3 + \eta M \ , \quad b = m_3 + (1 - \eta)M \ ,$$

$$c = m_1 + \xi M_3 \ , \quad d = m_2 + (1 - \xi)M_3 \ ,$$

$$q_R = q_1 + iq_2 \ , \quad q_L = q_1 - iq_2 \ , \tag{54}$$

$$K_R = K_1 + iK_2 \ , \quad K_L = K_1 - iK_2 \ .$$

The momentum wave function can be chosen as a function of M to fulfill the requirements of spherical and permutation symmetry. The S-state orbital function $\phi(M)$ is approximated by either

$$\phi(M) = N \exp\left[-\frac{M^2}{2\beta_G^2}\right] \quad \text{or} \quad \phi(M) = \frac{N'}{(M^2 + \beta^2)^{3.5}} \ , \tag{55}$$

which depend on two free parameters, the constituent quark mass and the confinement scale parameter β. The first function is the conventional choice used in spectroscopy, but it has a too strong falloff for large values of the four-momentum transfer. We use Schlumpf's parameters β_G=0.56 GeV, β=0.607 GeV, and the constituent quark mass, m_i=0.267 GeV.

The total wave function for the proton is given by

$$p = \frac{-1}{\sqrt{3}} \left(uud\chi^{\lambda 3} + udu\chi^{\lambda 2} + duu\chi^{\lambda 1} \right) \phi \ , \tag{56}$$

with

$$\chi_\uparrow^{\lambda 3} = \frac{1}{\sqrt{6}}(\downarrow\uparrow\uparrow + \uparrow\downarrow\uparrow - 2\uparrow\uparrow\downarrow),$$

$$\chi_\downarrow^{\lambda 3} = \frac{1}{\sqrt{6}}(2\downarrow\downarrow\uparrow - \downarrow\uparrow\downarrow - \uparrow\downarrow\downarrow) \ . \tag{57}$$

The spin wave functions $\chi^{\lambda 2}$ and $\chi^{\lambda 1}$ are the appropriate permutations of $\chi^{\lambda 3}$. The spin-wave function of the ith quark is given by

$$\uparrow = \mathcal{R}_i \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{and} \quad \downarrow = \mathcal{R}_i \begin{pmatrix} 0 \\ 1 \end{pmatrix} \ . \tag{58}$$

We now turn to the calculation of the proton form factors. The electromagnetic current matrix element can be written in terms of two form factors taking into account current and parity conservation:

$$\langle N, \lambda' p' \,| J^\mu |\, N, \lambda p \rangle = \bar{u}_{\lambda'}(p') \left[F_1(Q^2)\gamma^\mu + \frac{F_2(Q^2)}{2M_N} i\sigma^{\mu\nu}(p' - p)_\nu \right] u_\lambda(p) \tag{59}$$

with momentum transfer $Q^2 = -(p' - p)^2$ and $J^\mu = \bar{q}\gamma^\mu q$. For $Q^2 = 0$ the form factors F_1 and F_2 are respectively equal to the charge and the anomalous magnetic moment in units e and e/M_N, and the magnetic moment is $\mu = F_1(0) + F_2(0)$. The Sachs form factors are defined as

$$G_E = F_1 - \frac{Q^2}{4M_N^2} F_2 \,, \quad \text{and} \quad G_M = F_1 + F_2 \,, \tag{60}$$

and the charge radii of the nucleons are

$$\left\langle r_i^2 \right\rangle = -6 \frac{dF_i(Q^2)}{dQ^2}\bigg|_{Q^2=0} \,, \quad \text{and} \quad \left\langle r_{E/M}^2 \right\rangle = -\frac{6}{G_{E/M}(0)} \frac{dG_{E/M}(Q^2)}{dQ^2}\bigg|_{Q^2=0} . \tag{61}$$

The form factors can be expressed in terms of the $+$ component of the current:

$$F_1(Q^2) = \frac{1}{2P^+} \left\langle N, \uparrow \left| J^+ \right| N, \uparrow \right\rangle \,,$$

$$\tag{62}$$

$$Q_\perp F_2(Q^2) = -\frac{2M_N}{2P^+} \left\langle N, \uparrow \left| J^+ \right| N, \downarrow \right\rangle \,.$$

The form factors are calculated from the diagrams of Fig.8, using the "good" current J^+ so that no terms with $q\bar{q}$ pairs are involved. Schlumpf's result is

$$F_1(Q^2) = \frac{N_c}{(2\pi)^6} \int d^3q\, d^3K \left(\frac{E_3' E_{12}' M}{E_3 E_{12} M'} \right)^{1/2} \phi^\dagger(M')\phi(M)$$

$$\times \sum_{i=1}^{3} F_{1i} \left\langle \chi_\uparrow^{\lambda i} | \chi_\uparrow^{\lambda i} \right\rangle$$

$$\tag{63}$$

$$Q_\perp F_2(Q^2) = -2M_N \frac{N_c}{(2\pi)^6} \int d^3q\, d^3K \left(\frac{E_3' E_{12}' M}{E_3 E_{12} M'} \right)^{1/2} \phi^\dagger(M')\phi(M)$$

$$\times \sum_{i=1}^{3} F_{1i} \left\langle \chi_\uparrow^{\lambda i} | \chi_\downarrow^{\lambda i} \right\rangle$$

with $i = (uud)$ for the proton. Here the prime indicates the absorption of the momentum transfer as $K'_\perp = K_\perp + \eta Q_\perp$ and $q'_\perp = q_\perp$. The factors F_{1u} and F_{1d} are the charges of the u and d quarks. We also consider $G_A(Q^2)$. We take the hadronic axial-vector current to be

$$A_\mu = \bar{u}\gamma_\mu\gamma_5 d \ . \tag{64}$$

The form factor of interest is given by

$$2P^+ G_A(Q^2) = \left\langle B', \uparrow \left| A^+ \right| B, \uparrow \right\rangle \ . \tag{65}$$

The final step is to specify the operator r^2 for this relativistic model. We use

$$r^2 = 3(\rho^2 + \lambda^2) \tag{66}$$

where $\vec{\rho}$ and $\vec{\lambda}$ are canonically conjugate to the momenta $\vec{q}$ and $\vec{K}$:

$$\vec{\rho} = \frac{\vec{r}_1 - \vec{r}_2}{\sqrt{2}} \qquad \vec{\lambda} = \frac{2}{\sqrt{6}} \left(\xi \vec{r}_1 + (1-\xi)\vec{r}_2 - \vec{r}_3 \right) . \tag{67}$$

Note that $\vec{\rho}$ and $\vec{\lambda}$ reduce to the usual three-body variables in the non-relativistic limit of $\xi \to \frac{1}{2}$.

VI. RESULTS

It is worth while to begin by discussing whether or not point-like configurations occur in the relativistic models we employ. We do this by defining a quantity $r^2(Q^2)$ as

$$r^2(Q^2) \equiv \frac{\langle N, \uparrow | r^2 J^+ | N, \uparrow \rangle}{\langle N, \uparrow | J^+ | N, \uparrow \rangle}$$
$$= \frac{N_c}{(2\pi)^6 F_1(Q^2)} \int d^3q d^3K \left(\frac{E'_3 E'_{12} M}{E_3 E_{12} M'} \right)^{1/2} \sum_{i=1}^{3} F_{1i} \left\langle \phi(M'), \chi_\uparrow^{\lambda i} | r^2 | \chi_\uparrow^{\lambda i}, \phi(M) \right\rangle. \tag{68}$$

This quantity should not be confused with the transverse size, $b^2(Q^2)$, used in studies of color transparency. The production of a point-like configuration at large momentum transfer by the electromagnetic current is signaled by the vanishing of the transverse size for large Q^2.

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Nevertheless, since we are interested here in the properties of a nucleon which is not necessarily moving at a large momentum with respect to the medium, it is $r^2(Q^2)$, as defined in (68), which enters our calculations. For comparison, we may define the quantity $b^2(Q^2)$ from (68) by including only a single transverse component of the operator r^2. We see from Fig. 8 that both the power-law and the Gaussian wave functions display significant reductions of the quantity $r^2(Q^2)$ for increasing Q^2, however only the power-law wave function is suppressed in the case of the transverse size, $b^2(Q^2)$. This behavior has been noted previously [26], and exemplifies the feature that power-law wave functions have point-like configurations in the transverse variables, but that such are absent in Gaussian wave functions. We shall use only the power law form, unless otherwise noted, because of its ability to reproduce data.

Comparisons between the free form factors and the medium modified ones are given in Figs. 10,11. The quantity G_E is suppressed at low momentum transfer, but G_M is not. This is due to the relativistic nature of our model. Indeed, the magnetic moment is increased by about 5%. This small change is not enough to cause disagreement with existing nuclear phenomenology. At higher momentum transfers both form factors are inhibited by about 10%. This is a significant effect, but not large enough to disagree with Jourdan's analysis [13].

This is more clearly seen by computing the nuclear response functions for the inclusive (e, e') cross section. (See for example [64].) The excitation energy is ω and the three momentum transfer is $\mathbf{q}$ so that

$$\frac{d^2\sigma}{d\Omega dE} = \sigma_M \Big[\frac{Q^4}{\mathbf{q}^4} R_L(\mathbf{q}, \omega) + \Big(\frac{Q^2}{2\mathbf{q}^2} + \tan^2\frac{\theta}{2} \Big) R_T(\mathbf{q}, \omega) \Big] \ , \tag{69}$$

with the Mott cross-section $\sigma_M = \alpha^2 \cos^2(\theta/2)/4E^2\sin^2(\theta/2)$. Here $Q^2 = -q_\mu^2 = \mathbf{q}^2 - \omega^2$ and θ is the scattering angle. The longitudinal R_L and transverse R_T response functions are calculated in the relativistic Fermi gas approximation at a density $\rho = 2k_F^3/3\pi^2$,

$$R_L = -\frac{2}{\pi\rho} \text{Im}(Z\Pi_{00}^p + N\Pi_{00}^n) \ , \tag{70}$$

$$R_T = -\frac{4}{\pi\rho} \text{Im}(Z\Pi_{22}^p + N\Pi_{22}^n) \ , \tag{71}$$

for a target with Z protons and N neutrons. (Note, $\mathbf{q}$ is assumed to be along the $\hat{\mathbf{1}}$ axis so the subscript 22 refers to a transverse direction.) Here the Fermi momentum is taken to be $k_F = 260$ MeV, which is appropriate for ^{56}Fe.

The polarization Π

$$\Pi^i_{\mu\nu}(q, \omega) = -i \int \frac{d^4p}{(2\pi)^4} \text{Tr}[G(p+q)\Gamma^i_\mu G(p)\Gamma^i_\nu] , \qquad (72)$$

is calculated with the nucleon Greens function $G(p)$

$$G(p) = \frac{p_\mu\gamma^\mu + M_\mathbf{p}}{2E_\mathbf{p}} \left[\frac{\Theta(k_F - |\mathbf{p}|)}{p_0 - E_\mathbf{p} - i\epsilon} + \frac{\Theta(|\mathbf{p}| - k_F)}{p_0 - E_\mathbf{p} + i\epsilon} \right] , \qquad (73)$$

and the electromagnetic vertex Γ, for $i = $ p (proton) or n (neutron). We assume the electromagnetic vertex has the form,

$$\Gamma^i_\mu = F^i_1\gamma_\mu + F^i_2\frac{i\sigma_{\mu\nu}q^\nu}{2M} \qquad (74)$$

even for off-shell nucleons in the medium. We show results comparing using F_i computed in free space and in the medium in Figs. 12 and 13.

It is immediately evident from Fig.12 that modifications to the magnetic form factor G_M have no effect on the longitudinal response, while the modifications to the electric form factor G_E lead to a suppression. This effect is due to the larger charge radius in the medium. The situation is quite different in the case of the transverse response shown in Fig.13. There it is seen that modifications to the electric form factor have no effect, while modifications to the magnetic form factor lead to only a minor suppression. In this case the increase in the magnetic moment, as shown in Fig.11, tends to cancel the effect of the increased radius.

We also study the effects of using the form factors of Figs. 10 and 11 in computing the $(e, e'p)$ cross sections for finite nuclei. In this case, absorption effects emphasize the role of the nucleon surface and the influence of color neutrality is about 60% smaller than shown in Figs. 12 and 13.

Our results for the medium modifications on the form factor G_A are shown in Fig.14. Once again there is about a 10% reduction. This effect could be observed in neutrino-nucleus scattering, in the (p, n) reaction, or parity violating electron scattering [65].

We now turn to computing the valence structure functions in the medium and in free space. One advantage of the light front formalism is that the wave function is closely related to the valence structure function F_2^{val}:

$$F_2^{val}(x_{Bj}) = \int d^3q d^3K \Psi^\dagger \Psi \delta \left(x_{Bj} - \frac{p_3^+}{P^+} \right). \tag{75}$$

Thus the operator $\mathcal{O}$ of Eq. (12) is the delta function which sets the plus-momentum of the quark equal to $x_{Bj}P^+$. This is meant to be taken at some momentum scale ~ 1 GeV. We may compute this quantity for the free and medium modified wave function. There are a host of other effects of the medium including Fermi motion, shadowing, pions in the medium, nuclear correlations, six quark bags, etc. (see the reviews [10].) Our concern here is to assess the effects of color neutrality. So we do not include these effects and do not compare our results with data.

The free version is shown in Fig.15, where the expected shape is obtained. The results for the ratio of medium modified to free structure functions using both the power law and Gaussian forms are shown in Fig.16. This shows that suppression of point like configurations does indeed lead to suppression of the valence structure function at large values of x_{Bj}. The normalization of the wave function ensures that the integral $\int dx_{Bj} F_2(x_{Bj})$ is unity whether the free or modified wave function is used. Thus one expects regions in which the ratio is bigger and smaller than unity. However, the result that there is suppression at large x_{Bj} is not a trivial consequence of kinematics and normalization as is seen by comparing with the Gaussian form. The Gaussian wave function does not display as much suppression at large x_{Bj}, which indicates the relative importance of large momentum components, or point-like configurations, in the power law versus Gaussian wave functions. The suppression occuring at large x_{Bj} is consistent with the relevant features of the data and therefore provides evidence for the existence of point like configurations.

VII. SUMMARY AND DISCUSSION

We have used the ideas of color neutrality to motivate a functional form of the central shell model potential:

$$V(r, R) = V_0 \rho(R) \frac{r^2}{< r^2 >}, \tag{76}$$

where $< r^2 > \equiv < \phi_0 | r^2 | \phi_0 >$ is the nucleonic expectation value of the operator r^2, $V_0 \approx -50 \text{MeV}$ and $\rho(R)$ is the nuclear density normalized so that $\rho(R = 0) = 1$. The concern here is with fluctuations, so that we rewrite the central shell model potential as

$$V(r, R) = V_0 \rho(R) + V_0 \rho(R) \quad \frac{r^2 - < r^2 >}{< r^2 >}, \tag{77}$$

and treat the second term as a perturbation. The effects of this perturbation can be evaluated, using Eq.(11) (or using the closure approximation of Eq.(12)) for any nuclear process.

Solving a set of toy models indicates that a perturbative treatment is valid and furthermore that a closure approximation may be used to avoid computing the complete spectrum of baryonic wavefunctions usually necessary in perturbation theory. The toy-model results are consistent with the notion that point like configurations can be suppressed in the nuclear medium.

Light front quantum mechanics, along with a specific model of the nucleon wave function [34–36], is next employed to compute the influence of medium effects at relatively high momentum transfer. We find that at low values of $Q^2 < 1 \text{ GeV}^2$ the electric form factor is suppressed and displays an increased charge radius, but that while the magnetic radius is also increased so is the magnetic moment. This leads to the result that the (e, e') transverse response, shown in Fig.13, is largely unaffected by the medium in this context. This behavior of the transverse response has previously been interpreted as signaling no change in the magnetic radius –contrary to the result obtained here. At higher values of Q^2 both form factors are suppressed in the medium as is G_A. These results are in accord with ideas about the suppression of the longitudinal response, but are not inconsistent with the analysis of Jourdan [13].

The results for the medium modifications of the F_2 structure function show a suppression at large values of x_{Bj} for both the power-law and Gaussian wave functions. However, the suppression is greater in the case of the power law, indicating the more pronounced role of high momentum components or point-like configurations there. This is consistent with the works of [1] and [26,27] and more importantly, provides evidence for the existence of point like configurations.

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FIGURES

FIG. 1. Possible configurations of the proton wave function.

FIG. 2. The matrix element of Eq. (12) (j_μ commutes with h_1^{rel}.)

FIG. 3. The influence of the attractive Coulomb potential in Eq.(40) on the wave function is illustrated.

FIG. 4. The influence of the medium on the wave function is illustrated as described in the text.

FIG. 5. The medium-modified exact wave function and that calculated in the closure approximation are plotted as a function of the relative coordinate, r.

FIG. 6. The exact medium-modified, free, and the closure-approximated form factors are plotted as a function of the square of the momentum transfer.

FIG. 7. Medium modified and free form factors calculated from the analytic expression given in Eq.(41) are plotted as a function of the square of the momentum transfer.

FIG. 8. The absorption of momentum by the valence quarks is illustrated.

FIG. 9. The quantities $r^2(Q^2)$ and $b^2(Q^2)$ for power-law and Gaussian wave functions.

FIG. 10. The free and medium modified electric form factors vs. the square of the four-momentum transfer, Q^2.

FIG. 11. The free and medium modified magnetic form factors vs. Q^2.

FIG. 12. The longitudinal response vs the energy transfer, ω.

FIG. 13. The transverse response vs. ω.

FIG. 14. The free and medium-modified axial vector form factors vs Q^2.

FIG. 15. Proton structure functions are plotted as a function of the scaling variable x_{Bj} for power-law and Gaussian wave functions.

FIG. 16. The ratio of the medium-modified to free structure function is plotted as a function of the scaling variable for the power-law and Gaussian wave functions.